changes in the shapes of the phase shifter characteristics. A more suitable ferrite or geometry should also reduce this error.

The effect of servo error signal in these measurements is negligible: the gain of the system is sufficient to insure errors of only 2 or 3 degrees.

Conclusion

A control loop has been placed around a ferrite phase shifter to permit precise phase settings. The frequency of a control signal determines these phase settings. The range of control frequencies required can be varied to suit the dispersive characteristics of the ferrite and the stability of the control signal sources. When the duty cycle of the signal whose phase is being shifted is large enough, a heterodyne method can be applied. In this case, the dispersive effects are eliminated as a possible source of error. Preliminary experimental data confirms the expected performance characteristics of the phase control system.

Tables for Cascaded Homogeneous Quarter-Wave Transformers*

LEO YOUNG†

Summary—**Quarter-wave transformers are frequently required in microwave and UHF systems. An exact design procedure is known but involves lengthy calculations. Faced with the design of many such transformers, the calculations were programmed on an IBM 704 digital computer. The speed of computation is such that several hundred designs for 2, 3, and 4 section transformers were systematically computed in a few minutes. The results are reproduced here in tables, which should permit the calculation of most cases of practical interest by interpolation.**

Introduction

MICROWAVE and UHF systems frequently require transformer sections to connect transmission lines of different characteristic impedances with minimum reflection. The multisection quarter-wave transformer serves this purpose well, but it is only comparatively recently that exact design procedures have been published. These are limited to homogeneous transformers, such as coaxial line or E-plane waveguide transformers.

Computation of Impedance Ratios

Design formulas resulting in an equal ripple or Tchebycheff-type response were given by Cohn,[1] Collin,[2]

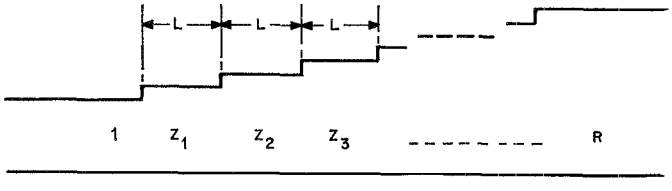

Fig. 1—Quarter-wave stepped transformer.

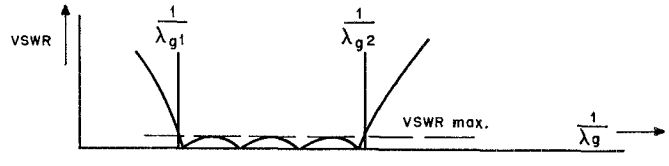

Fig. 2—Typical response curve.

and Riblet,[3] and were experimentally verified. As the computations are somewhat tedious, they were programmed on an IBM Type 704 electronic digital computer for transformers with 2, 3, and 4 sections. For the computation of the characteristic impedance ratios Z_1, Z_2, Z_3, $\cdots$ (Fig. 1), the computer program follows the exact method of Collin and Riblet. The length L of each section (Fig. 1) is nominally a quarter guide wavelength and is defined by

$$L = \frac{\lambda_{g1}\lambda_{g2}}{2(\lambda_{g1} + \lambda_{g2})} \tag{1}$$

where λ_{g1} is the longest and λ_{g2} is the shortest guide wavelength in the pass band (Fig. 2).

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† Westinghouse Electric Corp., and The Johns Hopkins University, Baltimore, Md.

[1] S. B. Cohn, "Optimum design of stepped transmission-line transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 16–21; April, 1955.

[2] R. E. Collin, "Theory and design of wide-band multisection quarter-wave transformers," Proc. IRE, vol. 43, pp. 179–185, February, 1955.

[3] H. J. Riblet, "General synthesis of quarter-wave impedance transformers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 36–43; January, 1957.

TABLE I
MAXIMUM VSWR FOR 2-SECTION QUARTER-WAVE TRANSFORMERS

Impedance Ratio, R	Bandwidth, W								
	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00	1.20
1.25	1.00	1.00	1.01	1.01	1.02	1.03	1.05	1.08	1.11
1.50	1.00	1.01	1.01	1.02	1.03	1.05	1.09	1.15	1.22
1.75	1.00	1.01	1.02	1.03	1.05	1.07	1.13	1.21	1.32
2.00	1.00	1.01	1.02	1.04	1.06	1.08	1.16	1.27	1.41
2.50	1.00	1.01	1.03	1.05	1.08	1.12	1.22	1.37	1.58
3.00	1.00	1.01	1.03	1.06	1.10	1.14	1.27	1.47	1.74
4.00	1.00	1.02	1.04	1.08	1.13	1.19	1.37	1.64	2.04
5.00	1.01	1.02	1.05	1.09	1.15	1.23	1.45	1.80	2.33
6.00	1.01	1.03	1.06	1.11	1.17	1.26	1.53	1.95	2.60
8.00	1.01	1.03	1.07	1.13	1.22	1.33	1.67	2.23	3.13
10.00	1.01	1.04	1.08	1.15	1.25	1.38	1.80	2.50	3.64
12.50	1.01	1.04	1.10	1.18	1.29	1.45	1.95	2.82	4.27
15.00	1.01	1.05	1.11	1.20	1.33	1.51	2.09	3.13	4.89
17.50	1.01	1.05	1.12	1.22	1.36	1.57	2.23	3.44	5.50
20.00	1.01	1.05	1.13	1.24	1.40	1.62	2.36	3.74	6.11
25.00	1.01	1.06	1.14	1.27	1.46	1.72	2.62	4.33	7.32
30.00	1.02	1.07	1.16	1.30	1.51	1.82	2.87	4.91	8.52
40.00	1.02	1.08	1.19	1.36	1.62	2.00	3.36	6.06	10.91
50.00	1.02	1.09	1.21	1.41	1.72	2.17	3.83	7.20	13.29
60.00	1.02	1.10	1.24	1.46	1.81	2.34	4.30	8.33	15.66
80.00	1.03	1.12	1.28	1.55	1.98	2.65	5.21	10.57	20.41
100.00	1.03	1.13	1.32	1.63	2.15	2.96	6.11	12.81	25.15

TABLE II
MAXIMUM VSWR FOR 3-SECTION QUARTER-WAVE TRANSFORMERS

Impedance Ratio, R	Bandwidth, W								
	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00	1.20
1.25	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.03	1.06
1.50	1.00	1.00	1.00	1.00	1.01	1.01	1.03	1.06	1.11
1.75	1.00	1.00	1.00	1.00	1.01	1.02	1.04	1.08	1.16
2.00	1.00	1.00	1.00	1.01	1.01	1.02	1.05	1.11	1.20
2.50	1.00	1.00	1.00	1.01	1.02	1.03	1.07	1.14	1.28
3.00	1.00	1.00	1.00	1.01	1.02	1.03	1.08	1.18	1.35
4.00	1.00	1.00	1.00	1.01	1.02	1.04	1.11	1.24	1.47
5.00	1.00	1.00	1.01	1.01	1.03	1.05	1.13	1.29	1.59
6.00	1.00	1.00	1.01	1.02	1.03	1.06	1.15	1.33	1.69
8.00	1.00	1.00	1.01	1.02	1.04	1.07	1.18	1.42	1.88
10.00	1.00	1.00	1.01	1.02	1.05	1.08	1.21	1.49	2.06
12.50	1.00	1.00	1.01	1.03	1.05	1.09	1.25	1.58	2.28
15.00	1.00	1.00	1.01	1.03	1.06	1.11	1.28	1.66	2.48
17.50	1.00	1.00	1.01	1.03	1.06	1.12	1.31	1.73	2.68
20.00	1.00	1.00	1.01	1.03	1.07	1.12	1.34	1.81	2.87
25.00	1.00	1.00	1.02	1.04	1.08	1.14	1.39	1.95	3.25
30.00	1.00	1.01	1.02	1.04	1.09	1.16	1.43	2.08	3.62
40.00	1.00	1.01	1.02	1.05	1.10	1.19	1.52	2.33	4.34
50.00	1.00	1.01	1.02	1.06	1.12	1.21	1.60	2.57	5.05
60.00	1.00	1.01	1.03	1.06	1.13	1.23	1.68	2.80	5.75
80.00	1.00	1.01	1.03	1.07	1.15	1.28	1.82	3.25	7.13
100.00	1.00	1.01	1.03	1.08	1.17	1.31	1.95	3.69	8.51

The bandwidth W is defined by

$$W = 2\left(\frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1} + \lambda_{g2}}\right). \tag{2}$$

Given the output-to-input impedance ratio R (taken greater than unity) and the bandwidth W, the computer calculates the relative characteristic impedances Z_1, Z_2, $\cdots$ of the intermediate sections for all three transformers ($n=2$, 3, and 4 sections).

TABLES

Tables I to III give the maximum VSWR in the pass band of the transformers for impedance transformations up to $R=100$ and bandwidths up to $W=1.20$ (or 120 per cent).

Tables IV to XIII give the values of the intermediate impedances Z_1 and Z_2. Only the lower impedances are given in the tables since the impedance ratios are symmetrical about the center. Thus the remaining impedances are obtained as follows:

For $n=2$,

$$Z_2 = R/Z_1.$$

For $n=3$,

$$Z_2 = \sqrt{R},$$
$$Z_3 = R/Z_1.$$

For $n=4$,

$$Z_3 = R/Z_2,$$
$$Z_4 = R/Z_1.$$

TABLE III
MAXIMUM VSWR FOR 4-SECTION QUARTER-WAVE TRANSFORMERS

Impedance Ratio, R	Bandwidth, W								
	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00	1.20
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.03
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.06
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.03	1.08
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.04	1.10
2.50	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.06	1.14
3.00	1.00	1.00	1.00	1.00	1.00	1.01	1.03	1.07	1.17
4.00	1.00	1.00	1.00	1.00	1.00	1.01	1.03	1.09	1.22
5.00	1.00	1.00	1.00	1.00	1.01	1.01	1.04	1.11	1.27
6.00	1.00	1.00	1.00	1.00	1.01	1.01	1.05	1.13	1.31
8.00	1.00	1.00	1.00	1.00	1.01	1.02	1.06	1.16	1.39
10.00	1.00	1.00	1.00	1.00	1.01	1.02	1.07	1.18	1.46
12.50	1.00	1.00	1.00	1.00	1.01	1.02	1.08	1.21	1.54
15.00	1.00	1.00	1.00	1.00	1.01	1.02	1.08	1.24	1.62
17.50	1.00	1.00	1.00	1.00	1.01	1.03	1.09	1.26	1.69
20.00	1.00	1.00	1.00	1.01	1.01	1.03	1.10	1.28	1.76
25.00	1.00	1.00	1.00	1.01	1.02	1.03	1.11	1.33	1.88
30.00	1.00	1.00	1.00	1.01	1.02	1.04	1.13	1.36	2.01
40.00	1.00	1.00	1.00	1.01	1.02	1.04	1.15	1.43	2.24
50.00	1.00	1.00	1.00	1.01	1.02	1.05	1.17	1.50	2.46
60.00	1.00	1.00	1.00	1.01	1.02	1.05	1.18	1.56	2.67
80.00	1.00	1.00	1.00	1.01	1.03	1.06	1.22	1.67	3.08
100.00	1.00	1.00	1.00	1.01	1.03	1.07	1.25	1.78	3.48

TABLE IV
MAXIMALLY FLAT QUARTER-WAVE TRANSFORMERS

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.88030	1.38110	1.17961	2.21803
1.25	1.05737	1.02829	1.01405	1.07223	15.00	1.96799	1.41512	1.19506	2.35186
1.50	1.10668	1.05202	1.02570	1.13512	17.50	2.04531	1.44475	1.20847	2.47169
1.75	1.15016	1.07255	1.03568	1.19120	20.00	2.11474	1.47108	1.22035	2.58072
2.00	1.18921	1.09068	1.04444	1.24206	25.00	2.23607	1.51650	1.24078	2.77447
2.50	1.25743	1.12177	1.05933	1.33204	30.00	2.34035	1.55498	1.25803	2.94423
3.00	1.31607	1.14793	1.07176	1.41051	40.00	2.51487	1.61832	1.28632	3.23492
4.00	1.41421	1.19071	1.09190	1.54417	50.00	2.65915	1.66978	1.30920	3.48136
5.00	1.49535	1.22524	1.10801	1.65686	60.00	2.78316	1.71340	1.32853	3.69752
6.00	1.56508	1.25439	1.12153	1.75529	80.00	2.99070	1.78522	1.36025	4.06810
8.00	1.68179	1.30219	1.14356	1.92323	100.00	3.16228	1.84359	1.38591	4.38263
10.00	1.77828	1.34089	1.16129	2.06509					

TABLE V
QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.10

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.88503	1.38341	1.18058	2.21984
1.25	1.05755	1.02843	1.01412	1.07230	15.00	1.97349	1.41772	1.19612	2.35396
1.50	1.10703	1.05227	1.02582	1.13526	17.50	2.05155	1.44761	1.20962	2.47405
1.75	1.15067	1.07290	1.03585	1.19140	20.00	2.12169	1.47417	1.22158	2.58333
2.00	1.18986	1.09113	1.04465	1.24231	25.00	2.24437	1.52002	1.24216	2.77755
2.50	1.25835	1.12238	1.05962	1.33240	30.00	2.34993	1.55888	1.25954	2.94775
3.00	1.31725	1.14868	1.07211	1.41097	40.00	2.52686	1.62290	1.28804	3.23925
4.00	1.41585	1.19171	1.09235	1.54482	50.00	2.67341	1.67495	1.31111	3.48642
5.00	1.49741	1.22645	1.10855	1.65767	60.00	2.79957	1.71911	1.33060	3.70327
6.00	1.56755	1.25578	1.12215	1.75625	80.00	3.01116	1.79188	1.36259	4.07510
8.00	1.68501	1.30391	1.14430	1.92448	100.00	3.18653	1.85108	1.38849	4.39079
10.00	1.78219	1.34289	1.16214	2.06660					

CORRECTION FOR STEP CAPACITANCES

Cohn[1] gave a method whereby the length of each section is usually reduced below a quarter wavelength for a first-order compensation of the shunt capacitances at the step discontinuities. This correction is in practice often negligible for coaxial line transformers, but worth making in many waveguide transformers. The waveguide case therefore was programmed for the computer using the formulas given by Marcuvitz[4] and Cohn.[1]

This program is available, but has not been used so far to compile tables, which would have to be somewhat

[4] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Series, vol. 10, pp. 307–310, McGraw-Hill Book Co., Inc., New York, N. Y.; 1951.

TABLE VI

QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.20

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.89934	1.39048	1.18351	2.22533
1.25	1.05810	1.02883	1.01431	1.07251	15.00	1.99014	1.42564	1.19936	2.36030
1.50	1.10808	1.05303	1.02619	1.13566	17.50	2.07045	1.45630	1.21313	2.48120
1.75	1.15218	1.07396	1.03636	1.19199	20.00	2.14275	1.48359	1.22534	2.59124
2.00	1.19181	1.09247	1.04530	1.24307	25.00	2.26955	1.53075	1.24635	2.78689
2.50	1.26113	1.12422	1.06049	1.33349	30.00	2.37903	1.57080	1.26411	2.95843
3.00	1.32079	1.15096	1.07317	1.41236	40.00	2.56334	1.63691	1.29328	3.25238
4.00	1.42080	1.19474	1.09373	1.54676	50.00	2.71681	1.69080	1.31691	3.50180
5.00	1.50366	1.23013	1.11019	1.66012	60.00	2.84956	1.73661	1.33690	3.72073
6.00	1.57501	1.26003	1.12402	1.75917	80.00	3.07359	1.81232	1.36974	4.09640
8.00	1.69473	1.30916	1.14656	1.92827	100.00	3.26067	1.87411	1.39637	4.41559
10.00	1.79402	1.34900	1.16472	2.07118					

TABLE VII

QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.30

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.92361	1.40261	1.18851	2.23468
1.25	1.05903	1.02952	1.01465	1.07286	15.00	2.01842	1.43925	1.20489	2.37110
1.50	1.10985	1.05431	1.02681	1.13635	17.50	2.10256	1.47128	1.21912	2.49336
1.75	1.15474	1.07577	1.03724	1.19298	20.00	2.17857	1.49982	1.23176	2.60471
2.00	1.19511	1.09476	1.04639	1.24436	25.00	2.31245	1.54927	1.25353	2.80281
2.50	1.26581	1.12736	1.06196	1.33533	30.00	2.42867	1.59139	1.27196	2.97663
3.00	1.32676	1.15486	1.07497	1.41472	40.00	2.62570	1.66118	1.30227	3.27478
4.00	1.42915	1.19992	1.09607	1.55006	50.00	2.79116	1.71830	1.32687	3.52805
5.00	1.51420	1.23642	1.11299	1.66428	60.00	2.93535	1.76704	1.34772	3.75057
6.00	1.58761	1.26731	1.12721	1.76413	80.00	3.18102	1.84798	1.38205	4.13285
8.00	1.71119	1.31815	1.15041	1.93470	100.00	3.38855	1.91438	1.40994	4.45808
10.00	1.81407	1.35949	1.16913	2.07896					

TABLE VIII

QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.40

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.95846	1.42039	1.19579	2.24816
1.25	1.06034	1.03051	1.01514	1.07337	15.00	2.05909	1.45924	1.21292	2.38669
1.50	1.11236	1.05616	1.02771	1.13733	17.50	2.14880	1.49328	1.22785	2.51095
1.75	1.15837	1.07839	1.03849	1.19441	20.00	2.23019	1.52371	1.24111	2.62419
2.00	1.19979	1.09808	1.04796	1.24621	25.00	2.37439	1.57661	1.26400	2.82586
2.50	1.27247	1.13192	1.06409	1.33797	30.00	2.50046	1.62184	1.28341	3.00301
3.00	1.33526	1.16050	1.07757	1.41810	40.00	2.71614	1.69719	1.31541	3.30730
4.00	1.44105	1.20746	1.09947	1.55479	50.00	2.89921	1.75924	1.34146	3.56620
5.00	1.52925	1.24557	1.11704	1.67026	60.00	3.06024	1.81246	1.36359	3.79399
6.00	1.60563	1.27790	1.13183	1.77127	80.00	3.33788	1.90144	1.40014	4.18597
8.00	1.73475	1.33128	1.15600	1.94397	100.00	3.57565	1.97500	1.42993	4.52011
10.00	1.84281	1.37482	1.17553	2.09018					

TABLE IX

QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.50

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	2.00472	1.44469	1.20561	2.26621
1.25	1.06205	1.03184	1.01579	1.07405	15.00	2.11313	1.48661	1.22379	2.40759
1.50	1.11564	1.05865	1.02891	1.13863	17.50	2.21034	1.52349	1.23966	2.53454
1.75	1.16311	1.08192	1.04017	1.19631	20.00	2.29897	1.55656	1.25378	2.65036
2.00	1.20593	1.10254	1.05007	1.24867	25.00	2.45707	1.61431	1.27820	2.85686
2.50	1.28121	1.13805	1.06694	1.34149	30.00	2.59642	1.66397	1.29897	3.03853
3.00	1.34643	1.16811	1.08106	1.42260	40.00	2.83728	1.74725	1.33332	3.35119
4.00	1.45672	1.21763	1.10402	1.56111	50.00	3.04417	1.81638	1.36140	3.61779
5.00	1.54908	1.25796	1.12249	1.67823	60.00	3.22797	1.87610	1.38531	3.85278
6.00	1.62941	1.29227	1.13804	1.78079	80.00	3.54879	1.97678	1.42497	4.25810
8.00	1.76591	1.34913	1.16353	1.95635	100.00	3.82733	2.06088	1.45744	4.60454
10.00	1.88089	1.39572	1.18416	2.10520					

TABLE X
QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.60

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	2.06334	1.47674	1.21838	2.28945
1.25	1.06418	1.03356	1.01663	1.07491	15.00	2.18171	1.52282	1.23794	2.43453
1.50	1.11973	1.06186	1.03045	1.14029	17.50	2.28850	1.56355	1.25505	2.56499
1.75	1.16904	1.08646	1.04233	1.19872	20.00	2.38640	1.60023	1.27032	2.68415
2.00	1.21360	1.10830	1.05278	1.25180	25.00	2.56229	1.66464	1.29679	2.89698
2.50	1.29215	1.14600	1.07061	1.34597	30.00	2.71863	1.72040	1.31937	3.08459
3.00	1.36042	1.17799	1.08555	1.42834	40.00	2.99167	1.81471	1.35688	3.40825
4.00	1.47640	1.23087	1.10990	1.56917	50.00	3.22888	1.89378	1.38768	3.68503
5.00	1.57405	1.27412	1.12952	1.68843	60.00	3.44157	1.96266	1.41402	3.92957
6.00	1.65937	1.31105	1.14608	1.79299	80.00	3.81681	2.08004	1.45792	4.35264
8.00	1.80527	1.37253	1.17327	1.97225	100.00	4.14625	2.17928	1.49407	4.71550
10.00	1.92906	1.42320	1.19535	2.12449					

TABLE XI
QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 0.80

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	2.22139	1.57157	1.25503	2.35502
1.25	1.06979	1.03839	1.01896	1.07727	15.00	2.36672	1.63055	1.27870	2.51076
1.50	1.13051	1.07092	1.03477	1.14487	17.50	2.49938	1.68331	1.29952	2.65136
1.75	1.18469	1.09933	1.04839	1.20539	20.00	2.62224	1.73135	1.31821	2.78024
2.00	1.23388	1.12466	1.06039	1.26046	25.00	2.84580	1.81693	1.35085	3.01151
2.50	1.32117	1.16862	1.08093	1.35838	30.00	3.04734	1.89229	1.37895	3.21651
3.00	1.39764	1.20621	1.09820	1.44427	40.00	3.40499	2.02249	1.42613	3.57268
4.00	1.52892	1.26891	1.12650	1.59161	50.00	3.72073	2.13434	1.46537	3.87971
5.00	1.64084	1.32078	1.14944	1.71688	60.00	4.00711	2.23376	1.49930	4.15287
6.00	1.73970	1.36551	1.16889	1.82708	80.00	4.51833	2.40750	1.55659	4.62954
8.00	1.91107	1.44091	1.20106	2.01680	100.00	4.97177	2.55856	1.60451	5.04246
10.00	2.05879	1.50397	1.22738	2.17873					

TABLE XII
QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 1.00

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	2.43686	1.72651	1.31212	2.45476
1.25	1.07725	1.04567	1.02244	1.08072	15.00	2.61818	1.80797	1.34251	2.62727
1.50	1.14495	1.08465	1.04121	1.15155	17.50	2.78500	1.88193	1.36949	2.78393
1.75	1.20572	1.11892	1.05743	1.21515	20.00	2.94048	1.95013	1.39387	2.92830
2.00	1.26122	1.14966	1.07177	1.27316	25.00	3.22539	2.07364	1.43691	3.18917
2.50	1.36043	1.20344	1.09642	1.37665	30.00	3.48399	2.18447	1.47439	3.42234
3.00	1.44816	1.24988	1.11727	1.46778	40.00	3.94578	2.38028	1.53827	3.83169
4.00	1.60049	1.32837	1.15166	1.62490	50.00	4.35536	2.55256	1.59228	4.18880
5.00	1.73205	1.39428	1.17976	1.75926	60.00	4.72769	2.70860	1.63965	4.50975
6.00	1.84951	1.45187	1.20377	1.87804	80.00	5.39296	2.98700	1.72101	5.07692
8.00	2.05579	1.55057	1.24383	2.08385	100.00	5.98279	3.23420	1.79038	5.57534
10.00	2.23607	1.63471	1.27697	2.26079					

TABLE XIII
QUARTER-WAVE TRANSFORMERS OF BANDWIDTH 1.20

R	$n=2$	$n=3$	$n=4$		R	$n=2$	$n=3$	$n=4$	
	Z_1	Z_1	Z_1	Z_2		Z_1	Z_1	Z_1	Z_2
1.00	1.00000	1.00000	1.00000	1.00000	12.50	2.70282	1.97543	1.39926	2.60341
1.25	1.08650	1.05636	1.02743	1.08558	15.00	2.92611	2.09480	1.44053	2.80190
1.50	1.16292	1.10495	1.05049	1.16102	17.50	3.13212	2.20457	1.47753	2.98360
1.75	1.23199	1.14805	1.07051	1.22899	20.00	3.32447	2.30687	1.51129	3.15225
2.00	1.29545	1.18702	1.08829	1.29123	25.00	3.67741	2.49446	1.57156	3.45984
2.50	1.40979	1.25594	1.11902	1.40276	30.00	3.99798	2.66499	1.62476	3.73777
3.00	1.51179	1.31621	1.14519	1.50152	40.00	4.57017	2.97034	1.71689	4.23225
4.00	1.69074	1.41972	1.18876	1.67300	50.00	5.07697	3.24219	1.79614	4.67007
5.00	1.84701	1.50824	1.22475	1.82083	60.00	5.53691	3.49018	1.86661	5.06832
6.00	1.98768	1.58676	1.25579	1.95244	80.00	6.35680	3.93524	1.98958	5.78224
8.00	2.23693	1.72383	1.30817	2.18248	100.00	7.08181	4.33178	2.09615	6.41923
10.00	2.45663	1.84304	1.35208	2.38228					

more extensive than the impedance ratio tables, owing to the greater number of independent parameters. Also, graphs are available,[4] in this case to reduce the time of a calculation by hand.

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